MODELING FOREST STAND STRUCTURE USING SPATIAL INTERPOLATION METHODS

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Modeling Forest Stand Structure Using Spatial Interpolation Methods

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Abstract

This study compared four geostatistical methods of interpolation (ordinary kriging, residual kriging, cokriging, and disjunctive kriging) with two traditional estimation methods (polygon mapping, and inverse distance weighting). The six techniques were used to spatially interpolate the number of stems, basal area, and number of seedlings on 82 plots in a 121-hectare first-order forest watershed at the USDA Forest Service, Fraser Experimental Forest, Fraser, Colorado, USA. Secondary variables used for cokriging included elevation, a combined value for slope and aspect, and the normalized difference vegetation index (NDVI) from Landsat-TM satellite imagery. The comparison criterion was the mean square error (MSE) calculated by cross-validation. The performance of the estimation techniques was different between variables. Overall, however, cokriging performed the best, followed by polygonal mapping. Residual kriging with a first or second degree trend surface yielded, in general, better results than ordinary kriging. Inverse distance weighting was generally outperformed by the linear kriging methods. The nonlinear kriging method, disjunctive kriging, performed least well. These results indicate that the additional accuracy from spatially cross-correlated variables substantially improves the estimation capability of cokriging, as compared to the other methods for these data.

Introduction

Forest management requires the estimation and mapping of forest resources. For reasons of time and cost an exhaustive measurement of every individual tree in a forest is not feasible. Different sampling techniques have been applied to estimate forest population variables. Some of the most widely used sampling methods are simple random sampling, stratified random sampling, and systematic sampling. All of these sampling techniques assume independence among the sampling units. However, this assumption is rarely fulfilled in nature. Husch et al. (1982, p. 182) write: "*In biological populations, such as a forest, the components are rarely, if ever, arranged completely independent of each other, but instead, show a systematic or periodic variation from place to place.*" Despite the violation of the independence assumption, these traditional sampling techniques are commonly used in forest inventory. While traditional statistics assume independent data, geostatistics take a different approach by quantifying and modeling spatial dependence. Information gathered from forest sampling is traditionally mapped as polygons. This approach assumes that forest parameters are homogeneous within each polygon and change abruptly at boundaries. Many natural phenomena, however, change gradually over space. Spatial interpolation techniques like geostatistical methods, can be applied to represent forest stand structure as a continuously varying surface.

Underlying the geostatistical approach is the assumption that the values at positions near one another tend to be more similar than others farther apart (spatial autocorrelation). The geostatistical estimation method known as kriging was developed by the South African mining engineer Danie G. Krige (1951) and the French geomathematician Georges Matheron (1962, 1963). The advantage of the kriging technique is that it results in unbiased estimates with minimized estimation error variances at unsampled locations. Many different kriging methods have been developed over the years. Some of the most widely used methods include ordinary kriging, residual kriging, cokriging, and disjunctive kriging. Ordinary kriging requires a stationary mean in data values over the study area. Many natural phenomena, however, exhibit a change in average values over the area (called a trend). Residual kriging is a method that takes such local trends into account when computing the estimates. A trend surface analysis is used to remove a trend in the data prior to the kriging process. Cokriging utilizes multivariate information to determine estimates at unsampled locations by exploiting the cross-correlation between a variable of interest and a set of spatially cross-correlated secondary variables. All three kriging methods belong to the group of linear geostatistics. Nonlinear methods apply specific nonlinear transformations to the original data, and any arbitrary data distribution is thereby changed to a standard normal distribution. Disjunctive kriging is such a nonlinear estimation method. This technique also allows the calculation of conditional probabilities, that a variable is greater than, or less than, some prescribed cut-off or tolerance value.

Originally applied in the mining industry, kriging soon found its way into many other disciplines, notably soil science (Burgess and Webster 1980; Heuvelink and Bierkens 1992; Goovaerts and Journel 1995), hydrology (Kitanidis and Vomvoris 1983; Aboufirassi and Marino 1983), ecology (Orloci 1978; Robertson 1987; Rossi et al. 1992; Pohlmann 1993), atmospheric science (Lajaunie 1984; Hudson 1993), remote sensing (Rossi et.al. 1994; Stonge and Cavayas 1995), and forestry. The first people applying geostatistics for forestry purposes were Guibal (1973), followed by Marbeau (1976). Both used kriging in forest inventory applications. Bouchon (1979) estimated the surface area and conducted a structural analysis of forest stands using kriging. Ramirez-Maldonado (1988) presented and discussed the theory and methods of geostatistics in the context of forest inventory. Biondi et al. (1989) applied ordinary kriging to estimate stem diameters, basal area, and 10-year periodic basal area increments in an old-growth stand of southwestern ponderosa pine (*Pinus ponderosa* Dougl. *Ex.* Laws. *var. scopulorum*) in northern Arizona. The temporal variation of the three variables was evaluated at 10-year intervals from 1920 to 1990. Samra et al. (1989) estimated tree heights after 1, 2, and 3 years of growth in a plantation of Dharek (*Melia azedarach* Linn.) in India using universal kriging. A trend in the data was removed by a method called median polishing. Kriging was then conducted on the median-polished values. Mandallaz (1993) applied different geostatistical estimation methods (kriging with errors, mixed kriging, double kriging, and universal kriging) for double sampling schemes as they are used in combined forest inventories. Fouquet and Mandallaz (1993) merged ground measurements (stem density and basal area density) with information from aerial photos using cokriging. Höck et al. (1993) estimated the site indices in Kaingaroa Forest in New Zealand using ordinary kriging. Site index was defined as the top height (measured in meters) that the best growing trees in a stand attain at age 20 years. Holmgren and Thuresson (1995) applied ordinary kriging to predict total wood volume, hardwood volume, and two monetary measures ("inoptimality-losses" for thinning and clearcutting treatments) in a forest estate in northern Sweden. The kriging estimates were combined with estimates from image analysis of digital aerial photos. These combined estimates were used to allocate tactical treatment units in forest management planning. Gilbert and Lowell (1997) used ordinary kriging (and an interpolation method called area-stealing) to estimate forest volume from three relatively dense sampling schemata in a balsam fir - white birch (*Abies balsamea* L. - *Betula papyrifera* Marsh.) forest in Quebec, Canada. Indicator kriging and sequential Gaussian conditional simulation were used by Hershey (1997). The techniques were applied to provide an estimate of the distribution of ten tree species in Pennsylvania and a measure of uncertainty. Mowrer (1997) created maps of uncertainty for estimated areas of potential old-growth forest conditions in Fraser Experimental Forest, Colorado, applying a geostatistical simulation method (sequential Gaussian simulation). Old-growth conditions were based on mean stem diameter, percent crown cover, and mean age of overstory stems. Kallas (1997) created a hazard rating map of Armellaria

root disease in the Black Hills of South Dakota. She fitted a second-degree trend surface to her data including x- and y-coordinates, elevation, slope, aspect, and precipitation as independent variables. Kriging was applied to the residuals of the trend surface. Metzger (1997) used residual kriging and cokriging to model basal area, percent open canopy, and height of the understory vegetation on the Austin Cary Memorial Forest in northern Florida. The trend in the data was removed by fitting a first- and second-degree trend surface to the data using x- and y-coordinates and satellite data as the independent variables. Phillips et al. (1998) applied cokriging for estimating ozone exposure in southeastern forests as the primary variable and monthly data on anthropogenic emissions of nitrous oxides (NO_x), average daily maximum temperature, wind directional frequencies, and distance downwind from anthropogenic NO_x sources as auxiliary variables.

Many interpolation techniques have been widely and successfully used in various fields of natural resource assessment, including forestry. To date there has not been any study that has compared the various techniques in forested areas. Given their increasing importance, conducting such a comparative study is certainly warranted. The variables of interest chosen for the current investigation were the number of trees, the basal area, and the number of seedlings. Auxiliary variables included elevation, a combined value of slope and aspect, and normalized difference vegetation index (NDVI) from Landsat-TM satellite imagery. Data were available for the Lexen Creek watershed in the Fraser Experimental Forest, Fraser, Colorado. The four methods of kriging applied in this project were ordinary kriging, residual kriging, cokriging, and disjunctive kriging. For purposes of comparison, the traditional forest mapping technique (here called polygonal mapping, where the variable of interest is averaged over all sample points within a polygon), as well as inverse distance weighting (an interpolation technique where the weight for each adjacent sample point is inversely proportional to its distance from the point being estimated) were included.

Methods and Material

Spatial Interpolation Methods

Spatial interpolation techniques use nearby sample information to predict values of interest in unsampled areas. All interpolation methods involve weighted linear combinations of nearby sample values in order to provide estimates at unsampled locations:

[1] $$Z^*(x_0) = \sum_{i=1}^{n} \lambda_i \cdot Z(x_i)$$

where $Z^*(x_0)$ is the estimate of variable Z at the unsampled location x_0, $Z(x_i)$ is the value of variable Z at the sample location x_i, λ_i is a weight assigned to the value $Z(x_i)$, and n is the number of nearby sample

values. Different approaches to assigning the weights to the data values give rise to many different methodologies.

A simplistic approach to incorporate the information from nearby samples is by weighting them all equally, using their mean as the estimate. Polygonal mapping is a method where a spatially distributed variable of interest is averaged over all sample points within a sampling unit (polygon). External landscape features are used to delineate polygons. These are the themes drawn on most thematic maps of forests, soils, geology, vegetation, land use, etc. This technique assumes that important variation occurs at boundaries: the variation within each polygon is smaller than the variation between the polygons. This kind of mapping leads to a "stepped" model of the landscape as the polygons form a discontinuous surface of plateaus (Burrough 1986).

Inverse distance weighting is an interpolation technique that gives more weight to nearby samples and less weight to those that are further away. The weight for each sample is inversely proportional to its distance from the point being estimated (Isaaks and Srivastava 1989):

[2] $$Z^*_{IDW}(x_0) = \frac{\sum_{i=1}^{n} \frac{1}{d_i^p} \cdot Z(x_i)}{\sum_{i=1}^{n} \frac{1}{d_i^p}}$$

where $Z^*(x_0)$ is the estimated value at unsampled location x_0, $Z(x_i)$ is the sample point value at location x_i, d is the distance from the sample location to the point being estimated, p is the distance exponent, and n is the number of sample points. As one decreases p, the weights given to the samples become more similar. As one increases p, the individual weights become more dissimilar, with most weight given to the nearest sample. Traditionally, the most common choice for the inverse distance exponent is 2 (Isaaks and Srivastava 1989).

Geostatistical interpolation methods rest on the recognition that the spatial variation from place to place of many properties cannot be modeled by simple mathematical functions. However, there is some spatial structure, i.e. spatial dependence that can be described by a stochastic approach. This spatial structure, or spatial autocorrelation, is the dependence between data values: values located close together show greater similarity than values that are farther apart. The geostatistical estimation process consists of two parts: (1) the *structural analysis*: the investigation and modeling of the spatial correlation that characterize a regionalized variable. (2) The *kriging* methods: the estimation process using features from the structural analysis to define the weighting factors. A detailed description of the theory of geostatistics is beyond the scope of this paper. The interested reader is referred to David (1977, 1988), Journel and

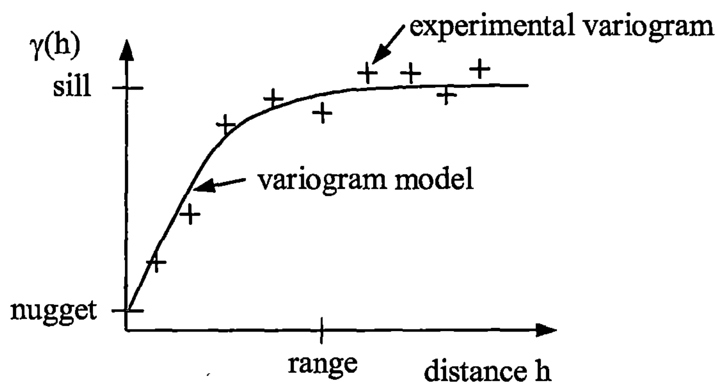
Huijbregts (1978), Isaaks and Srivastava (1989), Cressie (1991), Goovaerts (1997), Hohn (1998), and Armstrong (1998).

The spatial correlation of data values is assumed to vary with the distance between two points. The variogram is the basic geostatistical tool to measure how the spatial correlation changes with distance. The variogram value $\gamma(h)$ for a certain distance h can be estimated from the sample data:

[3] $$\gamma(h) = \frac{1}{2N(h)} \sum_{i=1}^{N(h)} \left[Z(x_i) - Z(x_i + h)\right]^2$$

where $Z(x_i)$ is the sample value of variable Z at position x_i, $Z(x_i+h)$ is the sample value of variable Z at position x_i+h, and N(h) is the number of pairs of observations separated by distance h. A plot of $\gamma(h)$ against h is known as the experimental semivariogram, or simply the variogram (Fig. 1). The variogram measures the average dissimilarity between data points that are separated by a distance h. The dissimilarity generally increases with increasing distance.

Fig. 1. The experimental variogram and the variogram model with its typical features: sill, range, and nugget.



A typical variogram shows three features: the sill, the range, and the nugget effect. As the separation distance between data points increases, the variogram values also increase. Finally, the variogram reaches a maximum value called the *sill*. The distance at which the variogram reaches the sill is called the *range*. At this distance there is no longer any spatial correlation between data points. Though the value of the variogram for distance zero is strictly 0, several factors, such as sampling error and short scale variability, may cause sample values separated by extremely small distances to be quite dissimilar. This results in a discontinuity at the origin of the variogram (a non-zero intercept) called the *nugget*. A mathematical model is fitted to the experimental variogram (Fig. 1). These variogram models must fulfill

a mathematical requirement known as positive-definiteness (Goovaerts 1997). The variogram models can be divided into two groups, namely models with a sill and models without a sill. Models with a sill are called *bounded models* (or transition models). The spherical, exponential and Gaussian models are examples of bounded variogram models. Models without a sill are called *unbounded models* (or non-transition models). The linear model is an example of an unbounded variogram model.

The variogram model is used to determine the weights for the geostatistical interpolation process. These kriging weights take into account the distance from the sample points to the point being estimated (the further away, the less weight) and the clustering of sample points (smaller weights for clustered samples). Furthermore, these weights assure that the kriging estimates are unbiased, i.e. that, on average, the difference between the predicted value and the actual value is zero. The kriging weights also assure that the error variance is minimized. This estimation variance is a statistical error measure defined to be the average squared difference between the predicted and the actual value. Because of these properties, ordinary kriging is also known as "best linear unbiased estimator" (BLUE). The estimation technique is "linear" because its estimates are weighted linear combinations of the available nearby sample data. It is "unbiased" since it tries to have the mean error equal to zero. It is "best" because it aims at minimizing the variance of the errors (Isaaks and Srivastava 1989). Since the error variances can be calculated and mapped, kriging provides a measurement of uncertainty as the confidence that can be placed in the estimates can be determined.

A serious restriction of ordinary kriging is the requirement of a stationary (i.e. constant) mean over the study area. Many natural phenomena, however, are known to be non-stationary. Ordinary kriging should not be used in the presence of a strong trend, a change in average value, as it yields erroneous and biased results. Residual kriging is a form of unbiased interpolation that takes account of local trends in the data when minimizing the error associated with the estimation process. Residual kriging consists of three steps: (1) a trend surface analysis (a polynomial regression to fit a polynomial surface by least squares through the data points) is applied to model the trend in the data. (2) Kriging is used on the residuals of the trend surface. And (3) the estimated residuals are combined with the trend surface to obtain the final estimates of the actual surface.

Cokriging utilizes multivariate information to improve estimates at unsampled locations. It is a method of unbiased estimation that minimizes the variance of the estimation errors by exploiting the spatial cross-correlation between the variable of interest and a set of secondary variables. Variograms for the primary and each secondary variable, as well as cross-variograms for each combination of these variables are used to determine the estimation weights. The cross-variogram measures how two different

variables vary jointly in space. The cross-variogram pairs values of different variables at different locations:

$$[4] \qquad \gamma_{ZW}(h) = \frac{1}{2N(h)} \sum_{i,j=1}^{N(h)} [Z(x_i) - Z(x_i + h)] \cdot [W(x_j) - W(x_j + h)]$$

where $\gamma_{zw}(h)$ is the cross-variogram value for primary variable Z and secondary variable W separated by distance h, $Z(x_i)$ is the value of the primary variable Z at position x_i, $Z(x_i+h)$ is the value of the primary variable Z at position x_i+h, $W(x_j)$ is the value of the secondary variable W at position x_j, $W(x_j+h)$ is the value of the secondary variable W at position x_j+h, and N(h) is the number of pairs of observations separated by distance h. Mathematical models are fit to the variograms and cross-variograms. The difficulty is that the models for the variograms and cross-variograms can not be built independently of one another. The linear model of coregionalization (Goovaerts 1997) provides a method for the combined modeling of the variograms and cross-variograms of all variables. The variograms and cross-variogram models are used to derive the cokriging weights for the primary and secondary variables. The cokriging estimate is a weighted linear combination of primary and secondary data values and is given by:

$$[5] \qquad Z^*_{CK}(x_0) = \sum_{i=1}^{n} \lambda_i^Z \cdot Z(x_i) + \sum_{j=1}^{m} \lambda_j^W \cdot W(x_j)$$

where $Z(x_i)$ is the value of the primary variable Z at position x_i, $W(x_j)$ is the value of the secondary variable W at position x_j, λ_i^Z is a weight assigned to the value of primary variable $Z(x_i)$, and, λ_j^W is a weight assigned to the value of secondary variable $W(x_j)$. These cokriging weights ensure unbiasedness and minimum error variances. Cokriging is a most useful estimation technique when the variable of interest is difficult or costly to sample, while a secondary, spatially correlated variable can be easily and inexpensively sampled.

All the geostatistical methods described so far belong to the field of linear geostatistics. These linear estimators may only be optimal when the variable under study has a normal distribution (Rendu 1980). In contrast, nonlinear geostatistical methods can be applied to variables that exhibit any non-normal distribution. Disjunctive kriging is an example for a nonlinear unbiased estimation method. It is better, theoretically, than linear estimators in providing minimum error variance estimates and exactness of estimation. The theory underlying disjunctive kriging is somewhat complex. The interested reader is referred to Kim et al. (1977), Rendu (1980) and Rivoirard (1994) who provide a detailed description of the method. In short: the original data values Z(x) with any arbitrary distribution are transformed into a new variable Y(x), that has a standard normal distribution with a mean of zero and unit variance. The variable to be estimated is then decomposed into a sum of uncorrelated (disjoint) components of sample values:

[6] $$Z_{DK}^{*}(x_0) = \sum_{k=0}^{K} C_k \cdot H_k^{*}[Y(x_0)]$$

where $H_k^*[Y(x_0)]$ represents the estimated value of the k^{th} Hermite polynomial at the estimation location x_0, and C_k are the Hermite coefficients. The $H_k^*[Y(x_0)]$ are estimated using the variogram model of the normalized data values and the k^{th} Hermite polynomial of the nearby sample values. The C_k are derived from the sample values using numerical integration. The optimal number k of Hermite polynomials and Hermite coefficients is found when the mean and variance of the original data, and the mean and variance of the retransformed data are approximately the same. In addition to the calculation of estimates and estimation variances, disjunctive kriging allows the calculation of conditional probabilities, that a variable is greater (or smaller) than some prescribed cut-off or tolerance value.

In a comparison between different interpolation techniques one would like to check the results of the different approaches and choose the estimation method that works best. Cross-validation uses only the information available in the sample data set (Isaaks and Srivastava 1989). One sample value is temporarily withheld from the sample data set. This value is then estimated using the remaining samples. This procedure is repeated for all samples and the resulting true and estimated values can then be compared. The differences between the estimated values Z^* and the true values Z are called residuals r:

[7] $$r = Z^*(x_0) - Z(x_0)$$

where $Z^*(x_0)$ is the estimated value at location x_0, and $Z(x_0)$ is the true value at that location. One measure of validation is the mean square error (MSE) which is calculated as:

[8] $$MSE = \frac{1}{n} \sum_{i=1}^{n} r^2$$

where n is the number of sample points. The smaller the MSE, the better the estimator.

Study Area

The Fraser Experimental Forest is a 93 square kilometer outdoor research laboratory maintained by the Rocky Mountain Research Station, USDA Forest Service. The forest is located approximately 160 kilometers northwest of Denver, Colorado, USA. The data for this study were collected on 82 1/125 hectare circular plots distributed across the Lexen Creek watershed, a 121 hectare first-order subalpine watershed in the Fraser Experimental Forest. Native vegetation is typical of the subalpine forest zone of the central Rocky Mountain: Engelmann spruce (Picea Engelmannii, Parry) and subalpine fir (Abies lasiocarpa, Nutt.) are the predominant trees at higher elevations, on north slopes, and along streams.

Lodgepole pine (Pinus contorta, Dougl.) is the predominant tree species at lower elevations and on drier upper slopes. Scattered patches of aspen occur in areas opened up by logging or fire. Occasionally, a large, old (450 to 500 years) Douglas fir (Pseudotsuga mentziesii) can be found (Alexander et al. 1985). Measurements from all plots reflect uneven-aged overstory conditions (Mowrer 1997). By all indicators mortality has only been caused by natural processes (Mowrer ibid.). Topography of the Experimental Forest is typical of Southern Rocky Mountain Province. The west side of the forest is characterized by rugged mountains and narrow, steep-sided valleys filled with alluvium and glacial outwash. South and east sides of the forest are remnants of an old peneplain, dissected by mountain glaciers and characterized by long, gentle, relatively uniform slopes. The north side is a nearly level, broad valley dissected by St. Louis Creek and surrounded by rolling hills (Alexander et al. 1985). Climate is cool and humid with long, cold winters and short, cool summers. Average yearly temperature at Forest headquarters (2,740 meter elevation) is 0.5° C. Mean monthly temperature for January is -10° C, for July 13° C. Precipitation over the entire Experimental Forest averages about 50 to 75 centimeters, with nearly two-thirds falling as snow from October through May (Alexander et al. ibid.).

Data Set

Three sample plot variables were used in this research: number of trees, total basal area, and number of seedlings. The 82 circular sample plots had a radius of 5 m. Plot center locations were spatially georeferenced using a six-channel global positioning system (GPS) receiver, and were subsequently differentially corrected to obtain a nominal 5 m r.m.s. (root mean square error) locational accuracy. All stems greater than 2.54 cm within the plot were counted and measured for dbh (diameter at breast height), from which the total basal area was calculated for the plot. Seedling numbers were derived by establishing a line transect across the plot on the slope contour. All seedlings within 45 cm of either side of the line were recorded.

The Landsat-TM satellite scene no. 93265028-01 (path 34, row 32) was acquired for 23rd September 1992. The scene was 100 percent cloudfree. The Lexen Creek study area covered a rectangular 267x167 30-meter pixel subscene of the full scene. The image was geometrically corrected using digital orthophotos (0.5 m pixel resolution) as the reference image. The normalized difference vegetation index (NDVI) was calculated using the following formula: (band4 – band3) / (band4 + band3). For each sample plot location the NDVI value was extracted.

Contour line coverages with 2 m elevation intervals were available for the study area and were used to create a digital elevation model (DEM). Elevation, slope and aspect maps were derived from the

DEM. For each sample plot location elevation, slope and aspect values were extracted. A combined slope/aspect value was computed for each sample point:

[9] $$\text{slope/aspect} = [\sin(\text{aspect}) + \cos(\text{aspect}) - 1] \cdot \tan(\text{slope})$$

where aspect is measured in radians, and slope is measured in degrees (Bonham et al. 1995; Stage 1976). Unfavorable, drier aspects (south and west) result in a negative value, whereas the favorable, wetter aspects (north and east) result in positive values. The steeper the slope, the more negative (more positive, respectively) the value.

Results and Discussion

Based on the 82 sample plots a total of 20,800 estimates were generated on a 10x10 m grid system. To evaluate each interpolation result cross-validation was applied. The residuals from this cross-validation procedure were used to compute a mean square error (MSE). Cross-validation was also applied to find the optimal number of nearest neighbors to include in each estimation process. Nearest neighbors ranging in numbers from 1 to 20 were tested. For each of three variables of interest (number of stems, total basal area, and number of seedlings) six interpolation methods were applied:

(1) Polygonal mapping: Traditional photointerpretation techniques were applied to delineate eight forest stand polygons on digital orthophotos and stereo pairs. An average value for each of the three variables of interest within each stand polygon was computed. The residuals were calculated as the deviation between each polygonal mean and each sampled value within that polygon.

(2) Inverse distance weighting: Two methods were applied: (a) inverse distance weighting calculated interpolated values by weighting sample points inversely proportional to their distance from the unsampled locations. (b) Inverse distance weighting squared weighted the sample points proportional to their squared separation distance from the point to be estimated.

(3) Ordinary kriging: Experimental variograms were calculated and a variogram model (spherical model, exponential model, Gaussian model, or power model) was fitted to the data. The models were used to determine the weights in the ordinary kriging process.

(4) Residual kriging: A first- and second-degree trend surface (using the x- and y-coordinates of the sample plot locations as the independent variables) was subtracted from the data values in order to remove a possible trend in the data. The experimental variograms for the residuals of the trend surface were calculated and a variogram model was fitted to the data. Ordinary kriging was conducted on the residuals. In the final step, the trend surface and the kriged residual surface were combined.

(5) Cokriging: Primary and secondary variables were available at all sample locations. For each primary variable all possible combinations of secondary variables (elevation, combined value of slope and aspect, and NDVI) were tested. Variograms for the primary variable and each secondary variable, as well as cross-variograms for each combination of these variables had to be computed.

(6) Disjunctive kriging: The original data were transformed to a normal distribution. The optimal number of Hermite polynomials and Hermite Coefficients was found when the mean and variance of the original data and the retransformed data were approximately the same. Experimental variograms were calculated using the transformed data.

The ISATIS (Geovariances 1999) and GEOPACK (Yates and Yates 1990) geostatistical software package, as well as the SPLUS (MathSoft 1998) statistical software package were used to conduct the different interpolation techniques and their cross-validations. The SPLUS functions used in this project are custom programs (Reich 1999), and are not part of the commercial product. The VARIOWIN (Pannatier 1996) variogram modeling software was used to display the experimental variograms and cross-variograms with their variogram and cross-variogram models. The SURFER (Golden Software Inc. 1994) graphics program was applied to display the interpolated maps and the variance of estimation.

Number of Stems

The first variable under investigation was the number of stems (STM). Table 1 shows the mean square error (MSE) for each of the interpolation methods. Cokriging was the best interpolation technique yielding the lowest MSE of 44.568. From all possible combinations of secondary variables, using the combined value of slope and aspect (SLOASP) as auxiliary variable yielded the smallest MSE. 20 nearest neighbors were included in the estimation process. The polygonal mapping technique (MSE of 44.906) was only slightly worse than cokriging. This showed that the spatial distribution of variable STM could be well described with discrete polygons. What is a possible explanation why cokriging (continuous surface) and polygonal mapping (discrete surface) performed similarly? Is the secondary variable slope/aspect picking up the same environmental trends as the delineation of the stand polygons? Slope and aspect play a strong role in snowpack accumulation, which provides the major water source during the growing season. North-facing and south-facing slopes have dramatically different vegetation types, density, and growth rates, which would have strongly influenced polygon delineation.

Table 1. Mean square errors (MSE) of eight interpolation methods for number of stems (STM).

Interpolation Method	MSE
Cokriging	44.568
Polygonal mapping	44.906
Residual kriging (2nd degree trend surface)	45.326
Residual kriging (1st degree trend surface)	46.804
Ordinary kriging	47.443
Inverse distance weighting squared	48.915
Inverse distance weighting	49.140
Disjunctive kriging	49.444

Polygonal mapping was followed in the ranking by residual kriging with a second-degree trend surface (MSE of 45.326), and residual kriging with a first-degree trend surface (MSE of 46.804). The removal of a potential trend in the data by using a first- and second-degree trend surface improved the estimation results in comparison with ordinary kriging (MSE of 47.443). Ordinary kriging outperformed inverse distance weighting squared (MSE of 48.915) and inverse distance weighting (MSE of 49.140). All kriging methods have the advantage that they take the interdependence of the sample points into account by giving less weight to clustered data points. The weights for inverse distance weighting, on the other side, solely depend on the distance from the sample points to the points being estimated. The nonlinear kriging method, disjunctive kriging, scored the lowest with a MSE of 49.444, despite the good results of the transformation process (mean and variance of the original and transformed data were similar, and the transformed data had a normal distribution).

The details of the cokriging process as the best interpolation method are summarized in Figure 2 and Table 2. The utilization of the spatial cross-correlation between the primary variable STM and secondary variable slope/aspect (SLOASP) helped to improve the estimation accuracy of variable STM. Figure 2 shows the experimental variograms of STM and SLOASP, as well as the cross-variogram for STM-SLOASP. The variogram model consisted of a combination of a spherical and a Gaussian model. Table 2 shows the specifications for both variogram models and the cross-variogram model.

Fig. 2. (a) Experimental variograms for primary variable number of stems per plot (STM) and (b) secondary variable slope/aspect (SLOASP), as well as (c) experimental cross-variogram with a spherical and a Gaussian (cross-) variogram model.

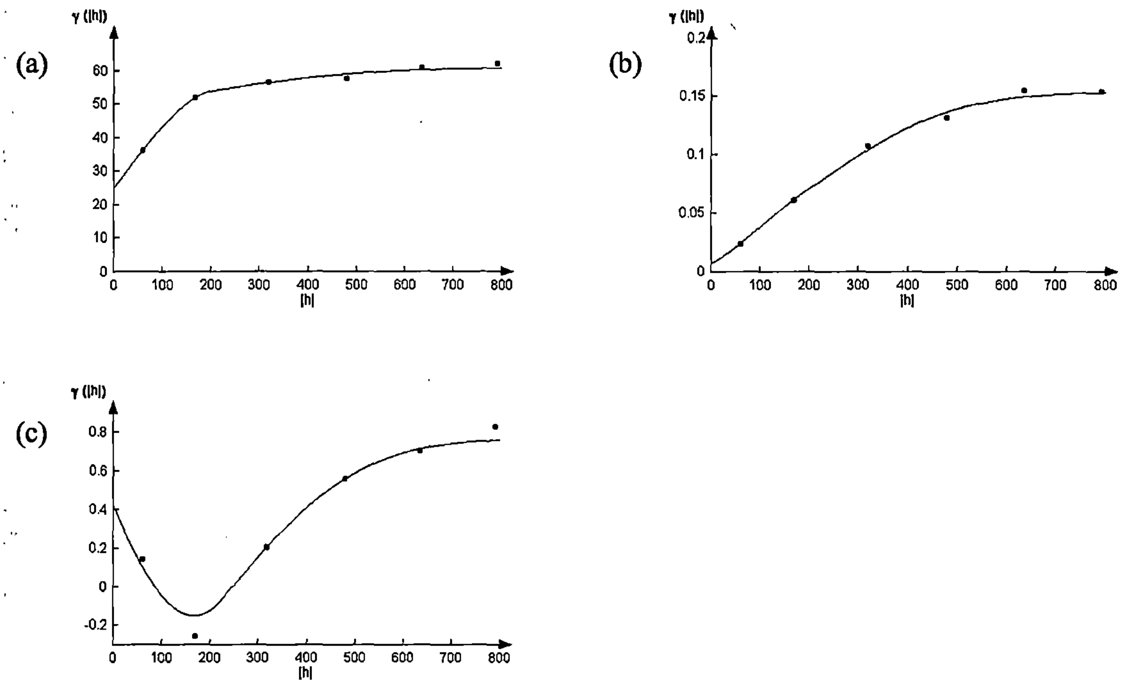
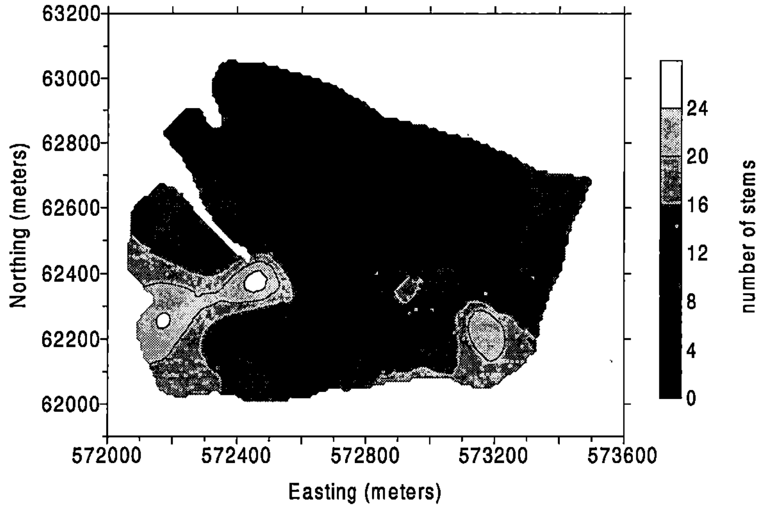


Table 2. Variogram and cross-variogram model specifications for primary variable number of stems per plot (STM) and secondary variable slope/aspect (SLOASP).

Variogram /	Spherical			Gaussian	
Cross-variogram	nugget	sill	range	sill	range
STM	25.456	23.682	200	13.189	650
SLOASP	0.007	0.034	200	0.117	650
STM-SLOASP	0.421	-0.897	200	1.242	650

A contour map of the estimates is displayed in Figure 3. The minimum value of the estimates was 1.30, and the maximum value was 27.20. Regions of high values of STM were concentrated in the southwest, and southeast corner of the study area, while low values occurred in the northern and central part of the area.

Fig. 3. Map of cokriging with primary variable number of stems (STM) and secondary variable slope/aspect (SLOASP).



Total Basal Area

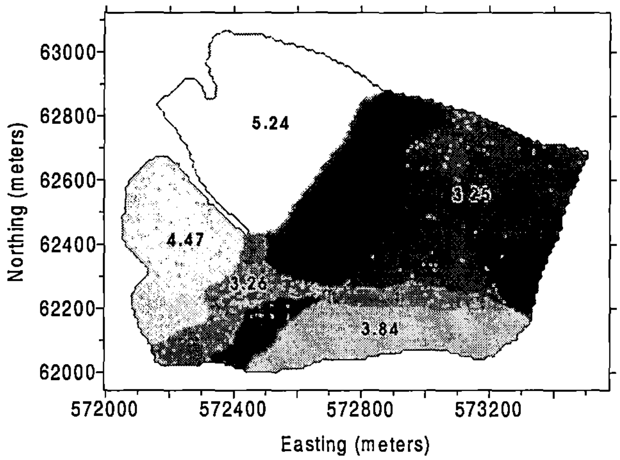
Table 3 shows the mean square errors (MSE) of all interpolation methods for total basal area (TBA). Polygonal mapping was the best interpolation technique with a MSE of 3.464. The spatial distribution of variable TBA was better described with discrete polygons than with any continuous surface. Residual kriging with a second-degree trend surface resulted in a MSE of 3.835. A second-degree trend surface was able to remove an underlying trend in the data.

Table 3. Mean square errors (MSE) of eight interpolation methods for total basal area per plot (TBA).

Interpolation Method	MSE
Polygonal mapping	3.464
Residual kriging (2nd degree trend surface)	3.835
Cokriging	3.910
Residual kriging (1st degree trend surface)	3.914
Ordinary kriging	3.951
Disjunctive kriging	3.993
Inverse distance weighting	4.122
Inverse distance weighting squared	4.598

Cokriging made good use of the spatial cross-correlation between the primary variable TBA and the auxiliary variable elevation, and reached a good estimation result with a MSE of 3.910. Applying a first-degree trend surface with residual kriging was less effective in modeling the underlying trend in the data than the second-degree trend surface: a MSE of 3.914 could be reached. Disjunctive kriging yielded a MSE of 3.993. The transformation from the original data to the normalized data performed well: the mean and variance of the original and transformed data were similar, and the transformed data had a normal distribution. All kriging methods reached very similar MSEs. Both inverse distance methods yielded the worst results: inverse distance weighting had a MSE of 4.122, and inverse distance weighting squared a MSE of 4.598. The reason for the superior performance of the kriging methods over the inverse distance weighting techniques lies again in the fact that the kriging methods take the clustering of the data points into account. Figure 4 shows the results of polygonal mapping as the best interpolation method. The maps shows the TBA per polygon. The polygon in the eastern part of the study area had the smallest value with 2.40, and the polygon in the northwest corner had the highest value with 5.24.

Fig. 4. Polygonal map of total basal area (TBA) showing the mean per polygon.



Number of Seedlings

The MSEs of all eight interpolation methods for the number of seedlings (SEEDL) are summarized in Table 4. The differences in the MSEs are much larger than for the interpolation results for variables number of stems (STM) and total basal area (TBA).

Table 4. Mean square errors (MSE) of eight interpolation methods for the number of seedlings per plot (SEEDL).

Interpolation Method	MSE
Inverse distance weighting squared	69.881
Cokriging	84.646
Inverse distance weighting	86.763
Ordinary kriging	95.903
Residual kriging (1st degree trend surface)	99.556
Residual kriging (2nd degree trend surface)	100.703
Polygonal mapping	113.338
Disjunctive kriging	118.975

The best result could be achieved with inverse distance weighting squared (9 nearest neighbors were included in the estimation process) having a MSE of 69.801, while inverse distance weighting was the third best method with a MSE of 86.763. All kriging methods require the sample data to have a normal distribution in order to be an unbiased and optimal interpolation method. Despite the fact that kriging has

often been found to be insensitive to lack of normality (Hohn 1998), the distribution of variable SEEDL, however, was far from a normal distribution (Fig. 5a). This lack of normality might be the reason for the bad results with ordinary kriging: the MSE was 95.903. But why is cokriging the interpolation method with the second best outcome (MSE of 84.646)? This kriging method also suffers from non-normally distributed data. Goovaerts (1997) and Asli and Marcotte (1995) both report that interpolation results from cokriging can be improved if the primary and secondary variable have a large correlation coefficient. Asli and Marcotte (ibid.) write that a high correlation (correlation coefficients of 0.4 and above) between the principal variable and the secondary variable is important to get the maximum benefit from the information available on the secondary variable. The correlation coefficient between primary variable SEEDL and auxiliary variable elevation was 0.57. This relatively strong correlation between these two variables might have helped to improve the estimation results of cokriging over ordinary kriging. Both residual kriging methods yielded results worse than ordinary kriging and cokriging: residual kriging with a first-degree trend surface had a MSE of 99.556, whereas residual kriging with a second-degree trend surface had a MSE of 100.703. Zimmerman et al. (1999) point out that for some types of surfaces it is better to completely ignore the modeling of trend (as ordinary kriging does) then to model the trend inappropriately. This might be one of the reasons for the better performance of ordinary kriging over residual kriging. The results for polygonal mapping with a high MSE of 113.338 indicated that variable SEEDL was better described as a continuous surface than with discrete polygons. Seedling numbers change on a small scale, depending e.g., on openings in the forest canopy. The delineation of the forest stand polygons, however, is based on the large-scale variability. This might be an explanation for the better estimation results using a continuous surface, which can take small-scale variability into account. Disjunctive kriging resulted in the worst outcome: the MSE was 118.995. This result can easily be explained by looking at the poor transformation results: the "normalized" SEEDL data were far from being normally distributed (Fig. 5b). While the mean of the original data was relatively close to the mean of the transformed data, the difference in the variances of the original and the retransformed data was very large (Table 5).

Fig. 5. Original data distribution (a), and normalized data distribution (b) for number of seedlings per plot (SEEDL).

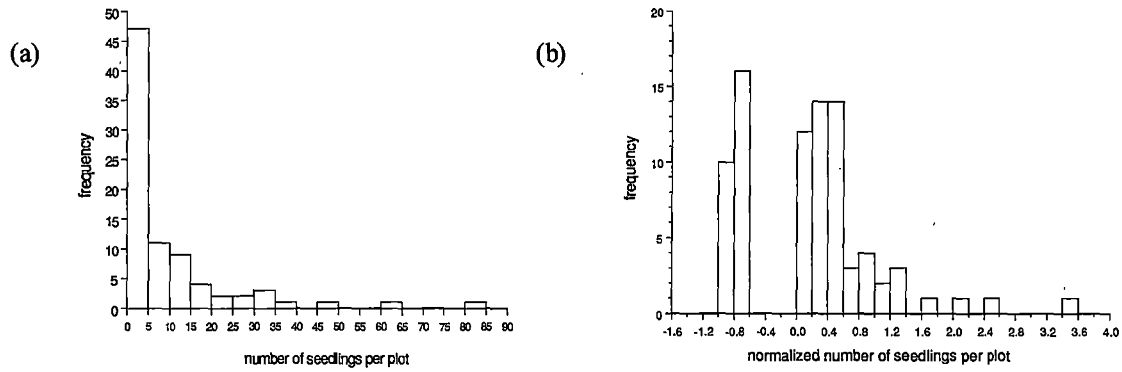
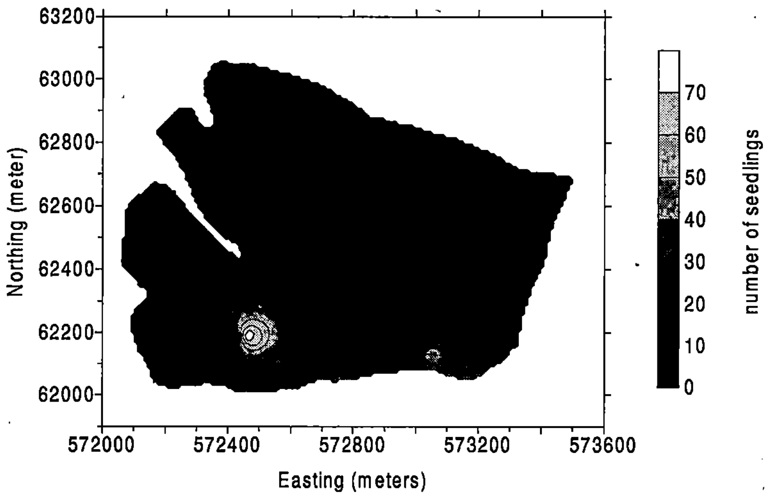


Table 5. Means and variances of original and retransformed data for number of seedlings per plot (SEEDL).

	Mean	Variance
Original data	9.13	191.82
Retransformed data	9.89	146.89

A contour map of the estimates for the best interpolation method (inverse distance weighting squared) is displayed in Figure 6. The minimum value of the estimates was 0.14, and the maximum value was 64.89. The highest number of seedlings could be found in the southwest part of the study area. Another zone of high values was found in the southeast corner, while a small area of medium values was located in the northeastern part. Zones of low numbers of seedlings covered the rest of the area.

Fig. 6. Map of inverse distance weighting squared with number of seedlings (SEEDL).



Summary

The performance of the different estimation techniques was different between variables. Table 6 gives a summary of the rankings for each interpolation method and each of the three variables under study. Column SUM is the sum of all rankings for each interpolation technique across all three variables.

Table 6. Summary of rankings for each interpolation method and each variable of interest: number of stems per plot (STM), total basal area per plot (TBA), and number of seedlings per plot (SEEDL).

Interpolation Method	STM	TBA	SEEDL	SUM
Polygonal mapping	2	1	7	10
Inverse distance weighting	7	7	3	17
Inverse distance weighting squared	6	8	1	15
Ordinary kriging	5	5	4	14
Residual kriging (1st degree trend surface)	4	4	5	13
Residual kriging (2nd degree trend surface)	3	2	6	11
Cokriging	1	3	2	6
Disjunctive kriging	8	6	8	22

There was no single best interpolation method. Overall, however, cokriging performed the best, followed, with some distance, by polygonal mapping. Residual kriging with a second-degree trend surface, and

residual kriging with a first-degree trend surface yielded, in general, better results than ordinary kriging. Inverse distance weighting squared and inverse distance weighting were generally outperformed by the linear kriging methods. The nonlinear kriging method disjunctive kriging performed least well.

The spatial distribution of two of the variables (STM and TBA) could be well described with discrete polygons. On the other side, variable SEEDL could only be poorly described as a discrete surface. *Polygonal mapping* assumes that important variation occurs at boundaries; the variation within each polygon is smaller than the variation between the polygons.

The quality of *inverse distance weighting* is affected by the clustering in the data. Inverse distance weights depend only on the distances between the data locations and the particular location to be estimated. The weights do not take into account the clustering of sample values as do the kriging methods. However, in cases where the prerequisites of kriging are not fulfilled (approximately normal distribution of the data), as it was the case with variable SEEDL, inverse distance weighting can yield better results than the kriging methods.

In theory, *ordinary kriging* is an optimal interpolator in the sense that it minimizes estimation variance when the variogram is known, the expected values of the mean and variance are constant over the study area, and the variable under study has a normal distribution. In practice, these conditions are never met (Weber and Englund 1992). However, as Weber and Englund (1992) note, kriging is robust: it generally produces good sub-optimal estimates even when reality departs from the ideal. Variables STM and TBA were slightly skewed to the right and ordinary kriging gave better estimation results than inverse distance weighting. Variable SEEDL, however, was highly skewed to the right and ordinary kriging gave worse results than inverse distance weighting. All kriging methods have the advantage that they, in addition to the distance from the sample points, take the interdependence of the sample points into account by giving less weight to clustered data points.

Residual kriging is a form of interpolation that takes trends into account when minimizing the estimation error. Removing a potential trend by applying a first- and second-degree trend surface improved the interpolation results for variables STM and TBA in comparison with ordinary kriging. For some types of surfaces it is better to completely ignore the modeling of a trend then to model the trend inappropriately. This might be the reason for the better performance of ordinary kriging over residual kriging for variable SEEDL.

By utilizing the spatial cross-correlation between primary and secondary variables the quality of the *cokriging* estimates was improved as compared to the results of the other kriging methods. Interpolation results from cokriging can be improved if the primary and secondary variable have a large

correlation coefficient. The correlation coefficient between primary variable SEEDL and auxiliary variable elevation was 0.57 and cokriging performed much better than ordinary kriging. On the other hand, primary variable STM and secondary variable slope/aspect had a correlation coefficient of 0.33, while primary variable TBA and secondary variable elevation had a correlation coefficient of 0.20. For both variables cokriging was only slightly better than ordinary kriging.

In general, *disjunctive kriging* performed least well. For variable SEEDL the transformed data were far from being normally distributed, a problem encountered when large numbers of identical data values are involved (Armstrong and Matheron 1986). The requirement of disjunctive kriging having transformed data with a normal distribution was therefore not fulfilled. But also for variables STM and TBA the estimation results were least satisfactory, despite the good results of the transformation process (mean and variance of the original and retransformed data were similar, and the transformed data had a normal distribution).

Conclusions

This study compared four geostatistical methods of interpolation (ordinary kriging, residual kriging, cokriging, and disjunctive kriging) with two traditional estimation methods (polygonal mapping, and inverse distance weighting). The six methods were used to spatially interpolate the number of stems, the total basal area, and the number of seedlings. The major conclusions of this study can be summarized as follows:

1) No single "best interpolation method" could be found. Each database required a new search for the best estimation technique. Weber and Englund (1992, p. 391) wrote: "The search for the ideal interpolator is far from over." This sentence is as true today as it was in 1992.

2) Ordinary kriging gave good estimation results as long as the data distribution was approximately normal. The method should not be applied in cases of strong deviations from a normal distribution. The advantage of all kriging methods lies in the fact that these techniques take the clustering of sample points into account.

3) Detrending the data with a first- and second-degree trend surface and applying kriging to the residuals gave good results. Residual kriging yielded better interpolation results than ordinary kriging for two of the three variables examined.

4) Cokriging was the interpolation method that performed very well across all data sets. Exploiting the spatial cross-correlation between a primary variable of interest and auxiliary variables resulted in better estimation results. A high correlation coefficient between the variables seemed to improve the

interpolation results. The disadvantage of cokriging is that the modeling of the variograms and cross-variograms has to be in compliance with the linear model of coregionalization. With increasing number of auxiliary variables the variogram modeling process becomes more difficult and cumbersome.

5) Disjunctive kriging should only be implemented if the retransformed data follow a normal distribution, and the mean and variance of the original and transformed data are similar. Because the results of this nonlinear kriging method were not better that the other kriging methods, and because of its increased computational requirements and complex analysis, the use of disjunctive kriging is not recommended for this type of data set. Additional information in the study area, in the form of auxiliary variables, seemed to be a more important consideration than whether an estimator is linear or nonlinear.

6) Inverse distance weighting techniques outperformed kriging methods only in cases where the requirements of kriging (approximately normal distribution of the data) were not fulfilled.

7) Polygonal mapping gave good estimation results for two of the variables under study. The spatial distribution of these two variables could be mapped very well with discrete polygons. Therefore, the representation of a variable as a continuous surface is not always appropriate.

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